SOCIALIST MATHEMATICS EDUCATION. Edited by Frank J. Swetz. 1978. Burgundy Press, Southampton, PA

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This book contains very little on *history* of any aspects of mathematics; its subject is education rather than history. The editor dedicated a few pages to sketching the influence of social structure on mathematical research in the past, and the various reporters concerned with mathematical education in the Soviet Union, The GDR, the People's Republic of China, Yugoslavia, Sweden, Hungary, and Tanzania restricted the historical aspect to a few lines or pages--the longest being that on Hungary. No specific philosophy of mathematics education is revealed in the various contributions, except perhaps in the Tanzanian one. Rather than the reality of mathematics education, the bureaucratic system is the main concern of the reporters.

GESCHICHTE DER ELEMENTARMATHEMATIK. BAND 1: ARITHMETIK UND ALGEBRA, 4th ed. By Johannes Tropfke. Edited by Kurt Vogel, Karin Reich, Helmuth Gericke. Berlin/New York; Walter de Gruyter, 1979. xiv + 742 pp. DM 184.

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"Tropfke" and "Cantor" (Moritz Cantor, *Vorlesungen über Geschichte der Mathematik*, 1900-1908) have been the encyclopedic surveys of the history of mathematics throughout the twentieth century. They were still to be consulted in the early stages of any historical study, even as they became less up to date. They had a number of common properties in addition to their almost biblical nature: namely, their tremendous wealth of references (the first three volumes of the third edition of Tropfke correspond to this new volume. They contained 3681 footnotes, mostly concise and bibliographical); their multivolume nature appearing over a substantial number of years; and the fact that their third editions were never completed. Their major differ-

ences were the concentration of Tropfke on "Elementarmathematik" and its topical organization whereas Cantor's work was chronological in arrangement, and included more advanced topics, especially in Volume 4, which was written by a group of his contemporaries.

The first edition of Tropfke was published in 1902 in two volumes. The last of the seven volumes of the second edition appeared in 1924. The fourth volume of the third edition was published posthumously in 1940, attended to by Kurt Vogel, who dated its forward "Kreigsweihnachten 1939." Vogel is now the senior editor of this new edition.

The publication of the third edition began in 1930. It was to have contained seven volumes with an index in the last volume. As a temporary measure each volume contained two tables, one mapping the page numbers of the second edition into those of the third edition, the other mapping the numbered footnotes of the second edition into those of the third edition. Thus, the index in Volume 7 of the second edition could be used with the third-edition volumes as they appeared.

The fourth edition has been so thoroughly reorganized that this arrangement would not be feasible. This is fortunate because its editors (who should perhaps be termed "authors" since so much has been rewritten, reorganized, extended, and added on) have devised a combination of bibliography and an index which, with an extended table of contents, guides a searcher well and easily through the book and into the literature.

This is done, in spite of the elimination of all footnotes, by concisely coded bracketed references to the bibliography and by cross-references within the bibliography. The sixty-three page bibliography contains approximately 1250 names in a single alphabetical listing of primary and secondary sources. With multiple listings under most names and the cross-referencing, it seems likely that this substantially exceeds the references in Volumes 1-3 (*Rechnen; Allgemeine Arithemetik; Proportionen, Gleichungen)* of the third edition with which this single volume is comparable. The remaining four volumes of the second-third edition *(Ebene Geometrie; Ebene Trigonometrie, Sphärik und Sphärische Trigonometrie; Analysis, Analytische Geometrie; Stereometrie Verzeichnisse)* will appear in the fourth edition as Band 2; *Geometrie;* and Band 3; *Analysis*.

A sampling of the literature list suggests that about one-quarter of the references are dated 1940 or later. A number of these additions to the original Tropfke are from Russian sources, and another noticeable group deals with Arabic materials. Needham's work on Chinese mathematics has also been appropriately included. In fact, this sampling-survey of Tropfke's literature list suggests again that a modern study of the shifts in the centers of both historical activity and historical interest would itself be interesting.

The fact that so much rewriting and reorganization has been done and that there are still two volumes to come make it difficult to identify the specific additions and deletions prepresented by the fourth edition. However, some general comments as well as specific details are possible. For example, there is less discussion of the solution of cubic and quartic equations, and a new section on the fundamental theorem of algebra has been added.

Under "Der Zahlbegriffe und seine Erweiterunge" there is a section on "Moderne Definitionen" (of cardinal numbers) which includes brief mentions of the work of Cantor, Dedekind, Peano, von Neumann, and others. This is followed by a section "Fachesprache" which traces some changes and variations in terminology and usage, a recurring new feature of this edition.

There is also a new section on algebraic and transcendental numbers, but old sections on number theory, the infinite, and the measurement of angles and time seem to have disappeared--some perhaps to reappear in later volumes.

The number of pages devoted to the approximate solution of equations is about the same in the third and fourth editions, but the latter gives less space to false position, mentions a wider range of methods, and notes the problem of improving approximation.

The chapter on applied arithmetic expands the discussion of "daily life" uses from that in the third edition and adds an extensive section of recreations.

In other words, there has been not only an inclusion of newer literature, but also some shift in the nature of the content to include new approaches to elementary concepts and a little more generality in some discussions. However, users must remember that the purpose and content of this book still stress *elementary* mathematics. For example, the short biographical notes in the literature list are useful but do not even hint at the work not related to this volume done by the authors cited. Naturally, this raises some question of the definition of "elementary." A brief discussion of groups and references to the book of Nový and Wussing are associated with the new section on the fundamental theorem of algebra, but no references to fields, rings, or abstract algebra in general were located.

A check was run on errors and omissions by following through a few topics. No errors were noted and only minor omissions were found. The latter were perhaps due to differences in choice or language familiariity between those of the editor-authors and the reviewer. For example, the index refers to several pages on binary numeration under "Dyadik" and the bibliography contains Anton Glaser's *History of Binary and Non-Decimal Numeration,* but neither is cross-referenced to the other and Glaser is not mentioned in the text or index. A person not knowing these connections would never make them other than by chance. Goldstine's

A History of Numerical Analysis from the 16th through the 19th Century (Springer-Verlag, Heidelberg and New York, 1977) is listed, but his earlier volume, *The Computer from Pascal to von Neumann* (Princeton University Press, 1972), which documents the introduction of the binary system into the design of computers, is not mentioned. Karl Menninger's *Zahlwort und Ziffer* is appropriately listed and used but no mention is made of the later English edition published by the MIT Press.

The first edition of Tropfke aimed to present a program for changing the school curriculum via a historical account. It was criticized for failing to use original sources. The second edition was improved through the assistance of Heinrich Wieleitner and Gustaf Eneström, and was sufficiently important to be the subject of a thesis giving additions to it (Mary Leontius Schulte, "Additions in Arithmetic, 1483-1700, to the Sources of Cajori's History of Mathematical Notation and Tropfke's Geschichte der Elementar Mathematik," University of Michigan, Ann Arbor; 1935). These results are incorporated into the new edition.

This new edition of Tropfke is the outstanding guide to the literature of the history of elementary mathematics. Many sections are prefaced with a short, select list of a few major sources or expositions. Where appropriate, sections conclude with detailed references to further examples. However, the book is much than a guide. Anyone who enjoys the history of mathematics can open it at random and be assured of an interesting collection of facts--more than mere tidbits--or read a few pages with a high probability of seeing a new connection as well as a new source. The first volume of the fourth edition of Tropfke has preserved the values of the earlier editions while eliminating much of their shortcomings as well as updating their coverage. Physically the fourth edition is a fine book, much improved over the third. It has a larger page, more open and readable modern type, and 138 pictures as well as many line drawings. Every reference library will need it as a resource, and any individual teaching the history of mathematics or doing historical research will covet it for his own working collection.